LS-14
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February 15, 1985

Light Source Injector Without e^+ Accumulator Ring

1. General Parameters of the Booster Synchrotron

Injection energy	0.300 GeV
Maximum energy	6 GeV
Magnet radius	25 m
Magnetic field:	
at injection	0.04 T
maximum	0.8 T
rise time	0.475 sec
closed orbit length	384 m
energy gain	15.36 keV/turn
Synchrotron radiation loss:	
at 0.3 MeV	28.7 eV
at 2.250 GeV	90.73 keV
at 6.0 GeV	4.6 MeV

2. Injector System

e^- linac:	
Energy	200 MeV
Current	2.5 A
Beam pulse length	22-25 nsec
e^+ linac:	
Energy	300 MeV
Current	8 mA
Beam pulse length	22 nsec
Number of positrons per bunch	1.1×10^9
Energy spread	±3.0 MeV

3. RF System

Energy range 0.03 GeV to 2.25 GeV - Cavity I	
Frequency	39.04 MHz
Harmonic number	50
Number of cavity	1
Maximum voltage per turn	200 kV
Min. over voltage $q = \frac{1}{\sin \phi_s}$	1.885
Energy range 2.25 GeV to 6.0 GeV - Cavity II	
Frequency	351.33 MHz
Harmonic number	450
Number of cavity	9 or 10
Maximum voltage per turn	6.5 MV
Minimum over voltage $q = \frac{1}{\sin \phi_s}$	1.413

4. Injection and Acceleration

The 1.1×10^9 positrons of the linac pulse will be injected in a constant magnetic field. The peak voltage of cavity I is 50 kV. Only 10 of the 50 buckets will be filled such that the bunch period is constant, approximately 128 nsec. When the injection is completed, the voltage is increased to 150 kV and the bunch length decreased to about 15 nsec. Taking into account the radiation damping, one can write the equation of the small amplitude oscillations in the form

$$\frac{e^2\phi}{dt^2} + 2\alpha_\varepsilon \frac{d\phi}{dt} + \Omega^2 \phi = 0 \tag{1}$$

where $\alpha_\varepsilon = \frac{c\, C_\gamma J_\varepsilon E^3}{4\pi R\rho} + \frac{1}{2E}\frac{dE}{dt}$,

$\Omega^2 = \left(\frac{c}{R}\right)^2 \left[\frac{h\eta eV \cos \phi_s}{2\pi E}\right]$ and $c_\gamma = 8.85 \times 10^{-5}$ m (GeV)$^{-3}$

Rather than using time as the independent variable, it is more convenient to use the energy E and Eq. (1) becomes

$$\frac{d^2\phi}{dE^2} + \left(2aE^3 + \frac{1}{E}\right)\frac{d\phi}{dE} + \frac{b}{E}\,\phi = 0 \qquad (2)$$

where $$a = \frac{c\,C_\gamma J_\varepsilon}{4\pi R\rho\,\frac{dE}{dt}} \text{ and } b = \left(\frac{c}{R\,\frac{dE}{dt}}\right)^2 \frac{h\eta\ eV\cos\phi_s}{2\pi}, \qquad (3)$$

and $\frac{dE}{dt} = ec\rho\,\frac{dB}{dt}$ (c = velocity of light). The frequency of phase oscillation Ω is much larger than the damping coefficient α_ε.

For constant value of $\frac{dB}{dt}$ and neglecting terms of first and higher order in $\frac{\alpha_\varepsilon}{\Omega}$, the solution of Eq. 2 can be written in the form

$$\phi = \phi_m(E)\cos 2\sqrt{bE}$$

where $$\phi_m(E) = \phi_m(E_o)\left(\frac{E_o}{E}\right)^{1/4} \exp.\ \frac{a}{4}\left(E_o^4 - E^4\right)$$

Noting that the bunch length $\tau = \frac{\phi_m}{\omega_{rf}}$ and using Eq. (3), one obtains

$$\frac{\tau}{\tau_o} = \left(\frac{E_o}{E}\right)^{1/4} \exp.\ \frac{c\,C_\gamma J_\varepsilon\left(E_o^4 - E^4\right)}{16\,\pi R\,\rho\frac{dE}{dt}}$$

Substitutions of τ_o = 15 nsec, E_o = 0.3 GeV, E = 2.25 GeV, c = 3 × 10^8, J_ε = 2, 2πR = 384 m, ρ = 25 m and $\frac{dE}{dt}$ = 12 GeV/sec $\left(\frac{dB}{dt} = 1.6 \text{ T/sec}\right)$ give τ = 2.07 nsec. This bunch length is small enough for the rf bucket of cavity II (f = 351.33 MHz, h = 450). The 1.1 × 10^9 positrons per bunch is much smaller than the threshold value for turbulence (mode mixing) longitudinal instability, so that no bunch lengthening will occur. Cavity II must be detuned during the acceleration with cavity I in order to reduce the induced voltage. To estimate the required detuning, it is assumed that the 1.1 × 10^9 positrons are uniformly distributed over the bunch length. Then, the harmonic components of the beam current are given by

$$I_k = \frac{2\ I_o \Delta t_o}{t_o} \frac{\sin\frac{\pi k \tau}{t_o}}{\frac{\pi k \tau}{t_o}} \tag{4}$$

where $I_o = 8$ mA, $\Delta t_o = 22$ nsec and $t_o = 128$ nsec is the bunch period. Only I_{45} will be able to excite cavity II

$$\Delta V_\pi = \frac{R_s I_{45}}{\left[1+\left(\frac{2Q\Delta f}{f}\right)^2\right]^{1/2}} \approx \frac{R_s I_{45}}{2Q\,\frac{\Delta f}{f}}$$

One sees from Eq. (4) that I_{45} and therefore ΔV_{II} is largest at the end of acceleration with cavity I. Setting $k = 45$ and $\tau = 2.07$ nsec, one finds $I_{45} = 1$ mA. Assuming $\frac{R_s}{Q} = 3$ kΩ gives $\Delta V_{II} = 1.5\ \frac{f}{\Delta f}$ volt. Setting $\Delta V_{II} = 1.5$ kV, one has $\frac{\Delta f}{f} = 10^{-3}$.

The detuning must be achieved by decreasing the resonance frequency of cavity II in order to prevent the excitation of Robinson instability. When the energy of the positron is 2.25 GeV, cavity I is turned off and cavity II on.

$$V_{II} = V_{II\ max}(1 - c^{-t/t_f}), \text{ where}$$

$$t_f = \frac{2Q}{\omega} = 40\ \mu\ \text{sec.}$$

The required voltage per turn of cavity II at the end of the transition from $h = 50$ to $h = 450$ is 1 MV. Substitution of this and $V_{II\ max} = 6.5$ MV gives $t = 6.7$ μsec. The decay time of cavity I is much longer ($\frac{2Q}{\omega} \approx 160$ μsec). Therefore, the phase of cavity I must be changed to either 180° or 0°. Detuning of cavity I may not be necessary.

The constant $\frac{dB}{dt}$ value of 1.6 t/sec implies a rise time of 0.475 sec. Assuming the same time to reduce the magnetic field to the injection value and 0.2 sec (50-Hz linac repetition rate) for injection, one obtains $\frac{dI}{dt} = 0.6$ mA/sec. To inject 300 mA, one will need 500 sec.